

The Elegance of Abstraction

Math homework 1

Set: Week 2, Due: Week 3

If you get stuck, you can talk to me, other students, and the peer math tutor. If you still don't understand after that, write down what you tried and what you didn't understand. I will give you some credit for this.

1. Give the next number two numbers in this sequence according to a pattern of your choice: $1, 5, \dots$? You should say what your pattern is. Remember, your answer is correct as long as it follows your chosen pattern.
2. Find the prime factorization of
 - (a) 28
 - (b) 36

Refer to our list of primes from class to remind yourself which numbers are prime.

3. Draw a Venn diagram as in class, for numbers up to 30, with the two circles representing
 - (a) left: numbers divisible by 3, right: numbers divisible by 5.
 - (b) left: numbers divisible by 2, right: numbers divisible by 4.

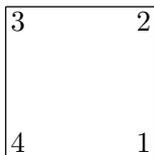
In each case describe the numbers in the intersection.

4.
 - (a) What is the largest prime number whose square is less than 150? In symbols this says: what is the largest prime number p such that $p^2 < 150$.
 - (b) Continue the Sieve of Eratosthenes up to 150. Note that you will need to check for divisibility by all prime numbers up to the one that was the answer to the previous part of the question. This should tell you all the prime numbers up to 150, and you should write these out in a list.
 - (c) Look up a list of prime numbers online and check that your list was the same.

Here's a grid you can use for this

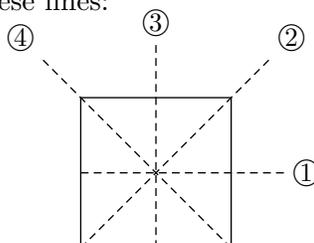
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150

5. In this question you are going to analyse the symmetries of a square just like we did for the equilateral triangle in class. Here is a square with its vertices labelled. It will help you to cut one out and put the numbers on the back as well, making sure they're on the same corners on the front and back.



The square has 8 symmetries. There are four rotations, by 90° , 180° , 270° and 0° which we will take anticlockwise. We will call these a_{90} , a_{180} , a_{270} and a_0 .

It also has four reflections, in these lines:



We will call these b_1, b_2, b_3, b_4 , so b_1 is reflection in the line marked 1, and so on.

- (a) Each of these symmetries permutes the vertices. Try it out and write down the resulting permutation in a grid, just as we did for the triangle. I have filled in a couple of lines for you so you can see if you're doing it right. Remember if the corner 1 goes to where 2 used to be, you write 2 in the table.

	1	2	3	4
a_0				
a_{90}	2	3	4	1
a_{180}				
a_{270}				
b_1	2	1	4	3
b_2				
b_3				
b_4				

- (b) Can you find of a permutation of 1, 2, 3, 4 that does not arise in this way? That is, is there a configuration of 1, 2, 3, 4 that the square will never get into? How is this different from the situation with the triangle?

6. Write down the converse of each of the following statements. For each part say whether the original statement is true or false, and also whether the converse is true or false. Look at the combinations of true and false that you get for the statements and their converses. Why have I chosen these four examples?

(a) If you live in Chicago then you live in Illinois.

(b) If you live in Chicago then you are American.

(c) If you are a mammal then you are a human.

(d) If you're taking Science 3123 then you're taking "The Elegance of Abstraction".