The Elegance of Abstraction
Math homework 3

Set: Week 8, Due: Week 9

If you get get stuck, you can talk to me, other students, and the peer math tutor. If you still don't understand after that, write down what you tried and what you didn't understand. I will give you some credit for this.

1. (a) Fill in this table for combining rotations of a square. Mark some patterns on it.

	0	90	180	270
0				
90				
180				
270				

   (b) Fill in this table for multiplication modulo 8 (as in the 8-hour clock). Mark some patterns on it. Does this have the same pattern as rotations of a square, or symmetries of a rectangle (that we did in class)?

$\times$	1	3	5	7
1				
3				
5				
7				

   (c) Fill in this table for addition modulo 4. Mark some patterns on it. Does this have the same pattern as rotations of a square, or symmetries of a rectangle?

$+$	0	1	2	3
0				
1				
2				
3				

2. Refer to your table for mulplication modulo 8 in part (b) above.

   (a) Find the identity element (the one such that multiplying by it does nothing).

   (b) Find an inverse for each element.

3. Is multiplication of real numbers associative? Give an example to show that division is not associative.

4. (a) Mark the following complex numbers on a complex plane: $1+2i, -2+i, -4-3i, 3-2i$.
   (b) What is the answer to the following additions?

$$\begin{aligned}(1+2i)+(3-2i) &= \\ (-2+i)+(3-2i) &=\end{aligned}$$

   (c) What is the answer to the following mulplications?

$$\begin{aligned}(1+2i)\times(3-2i) &= \\ (-2+i)\times(3-2i) &=\end{aligned}$$

Bonus questions: please turn in on separate paper.

5. (Bonus) Draw an 8×8 grid showing all the symmetries of a square and how they combine. List them in this order:

$$0,\ 90,\ 180,\ 270,\ b_1,\ b_2,\ b_3,\ b_4$$

   where the numbers are for rotations by those degrees, and the letters are for reflections. You should draw a square and label the lines of symmetry to show your system. When you've filled in the whole grid, draw in some patterns as we did in class for the equilateral triangle. Is this a product or a semi-direct product?

6. (Bonus) Suppose we've defined a binary operation $a \oplus b$ to mean $a + 2b$. We're now going to see if it's associative or not.

   (a) $1 \oplus 2$ is by definition $1 + (2 \times 2)$. What does this give?
   (b) $(1 \oplus 2) \oplus 3$ is by definition $x \oplus 3 = x + (2 \times 3)$ where x is your answer to the previous part. What does this give?
   (c) Now work out $2 \oplus 3$, and then $1 \oplus (2 \oplus 3)$. Is this the same as $(1 \oplus 2) \oplus 3$ or not? So is this binary operation associative or not?