The Elegance of Abstraction
Math homework 4

Set: Week 11, Due: Week 12

If you get get stuck, you can talk to me, other students, and the peer math tutor.

1. For each of the following functions $A \xrightarrow{f} B$ say if the function is injective, surjective, both or neither.

   (a) $A = \mathbb{N}$, $B = \mathbb{N}$, $f(x) = x + 1$ [this means that the function takes each number x as an input and adds 1 to produce $x + 1$ as output.]

   (b) $A = \{1, 2, 3\}$, $B = \{2, 4, 6\}$, $f(x) = 2x$.

   (c) $A =$ the set of all spring semester classes at SAIC. $B =$ the set of all SAIC classrooms. $f(x) =$ the classroom of x. [Assume that every class has only one classroom. I'm not sure if this is true or not.]

   (d) $A =$ the set of all SAIC classes that meet on Monday mornings. $B =$ the set of all SAIC classrooms. $f(x) =$ the classroom of x.

2. For each of the following relations, say if it is reflexive, symmetric, transitive.

   (a) World: students at SAIC. Relation: A is related to B if A is in a class with B.

   (b) World: people. Relation: A is related to B if they have the same birthday.

   (c) World: integers. Relation: A is related to B if A is a factor of B.

3. (a) Write down all possible functions from $A = \{1, 2, 3\}$ to $B = \{1\}$?

   (b) How many possible functions are there from $A = \{1, 2, 3\}$ to $B = \{1, 2\}$?

4. (a) Draw a lattice of factors of 42 like the one we did in class for factors of 30. Draw another for factors of 70. Make them have the same pattern.

   (b) Why do these lattices have the same pattern? Find another number whose lattice of factors looks the same.

5. Look at the square of arrows at the bottom of page 195. Find two squares like this in the category of trains at the bottom of page 198. Remember that some of the objects can be the same as each other (just because they're called A and B in the generic diagram doesn't mean they have to be different objects) and the arrows can be identity arrows.

Bonus questions: please turn in on separate paper.

6. (Bonus) Draw a lattice of factors of 210.

7. (Bonus) Consider functions $A \xrightarrow{f} B \xrightarrow{g} C$ and their composite $A \xrightarrow{g \circ f} C$.

   (a) If f is not injective can $g \circ f$ still be injective? What about if g is not injective?

   (b) If f is not surjective can $g \circ f$ still be surjective? What about if g is not surjective?

   Hint: try some small examples where A, B and C are sets with one or two elements, and use different combinations of functiosn f and g being injective and/or surjective. For example, try A with one element, B with two and C with one.