On the perfect quantity of cream for a scone

Eugenia Cheng
School of Mathematics and Statistics, University of Sheffield
E-mail: e.cheng@sheffield.ac.uk

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Abstract

We develop a formula for the perfect quantity of cream and jam for a scone. We prove that clotted cream is better than whipped cream, and show that the range of diameters for a good scone is 4cm–6cm.

1 Basic assumptions

Personal taste ratio. Everyone has their own personal perfect ratio of scone to jam to cream. We suggest 2:1:1 by *weight*. So if the scone weighs 80g, we need 40g of jam and 40g of cream. Note that this still means the cream will be much thicker than the jam—jam is denser than cream, so a thin layer of jam weighs the same as a thicker layer of cream.

Experimentally, we find that it's the weight of the cream that matters for the ratio, not the volume, so if you use whipped cream, you have to pile a lot more of it on to get a good taste. This is impractical, as we'll see.

Thickness of scone. A typical thickness for a scone is *half its diameter*. It seems that scones look nice if they're twice as wide as they are thick.

Construction of scone. We assume we're putting the jam on first and the cream on afterwards, because we like the sensation of cold cream squidging against the roof of our mouth. Building a good scone is like building a good sandcastle—you need a wider base, and then it needs to get narrower as it goes up so that it doesn't collapse (or drip). We assume a circular scone, with a rim of 5mm around the jam, and another rim of 5mm when you put the cream on.

2 Formulae

If r is the radius of the scone, then we have the following formulae for the thickness of the jam and the thickness of the cream. This is all in cm.

$$\text{thickness of clotted cream} = \frac{r^3}{8(r-1)^2}$$

$$\text{thickness of jam} = \frac{3r^3}{40(r-\frac{1}{2})^2}$$

To get the thickness of whipped cream, we have to multiply the clotted cream thickness by 1.5, because whipped cream is fluffier.

Here are some examples. The total thickness here is the total thickness of half a scone, piled with jam and cream.

diameter	thickness of scone	jam	clotted cream	total thickness	whipped cream	total thickness
8	2	0.39	0.88	3.28	1.33	3.73
7	1.75	0.36	0.86	2.96	1.29	3.39
6	1.5	0.32	0.84	2.67	1.27	3.09
5	1.25	0.29	0.87	2.41	1.30	2.85
4	1	0.27	1.00	2.27	1.50	2.77
3	0.75	0.25	1.69	2.69	2.53	3.53

The important thing to notice is that as the scone gets smaller the cream actually has to get thicker again. This is because we lose too much area around the edges when the scone is very small.

3 Conclusions

There are two other important criteria for how good a scone is.

1. Thickness of cream compared with scone: the cream should not be piled more thickly than the thickness of the scone, as then the whole thing is off balance.

2. Total thickness: we should be able to get it into our mouth. The natural opening width of the author's teeth is 2.8cm, so this is what the maximum total thickness of the adorned scone should be. This is a relaxed open width, so allows some space to open wide to fit the scone in. (Adjust this variable if you have a bigger or smaller mouth.)

Looking at the table, we can see that for clotted cream, the first criterion rules out the 3cm diameter as the clotted cream is thicker than the scone. The second criterion rules out diameters of 7cm and bigger as the total thickness is too big.

For the whipped cream, the first criterion rules out 5cm and smaller, because it makes the cream thicker than the scone. But the second criterion rules out 5cm and bigger, because the total thickness is then too big. This means that there is no good size of scone if you're going to use whipped cream.

In conclusion, we have proved the following result:

Theorem. *Clotted cream is better than whipped cream on a scone.*

4 Recommendations

Actually we recommend you pile as much jam and cream as you like onto your scone.

Appendix: calculations

We use the following variables; the right hand column gives the generic values we used in the formulae above, whereas the more general formulae below allow for differences in taste (i.e. different amounts of cream and jam).

r	=	radius of scone (half the diameter) in cm	*variable*
t	=	thickness, as fraction of diameter	$\frac{1}{2}$
d	=	density in g/cm^3	0.6
c	=	cream mass/scone mass ratio	0.5
j	=	jam mass/scone mass ratio	0.5
w	=	cream density in g/cm^3	clotted $= \frac{6}{5}$; whipped $= \frac{4}{5}$
b	=	jam density in g/cm^3	2

Then we have the following formulae.

Volume of half scone	=	$\pi r^3 t$
Mass of half scone	=	$\pi r^3 td$
Mass of cream on half scone	=	(scone mass) $\times\, c = \pi r^3 tdc$
Volume of cream	=	$\frac{\text{mass of cream}}{\text{density of cream}} = \frac{\pi r^3 tdc}{w}$
Area of cream	=	area of circle of radius $r - 1 = \pi(r-1)^2$
Thickness of cream	=	$\frac{\text{volume}}{\text{area}} = \frac{r^3 tdc}{w(r-1)^2}$
Mass of jam on half scone	=	(scone mass) $\times\, j = \pi r^3 tdj$
Volume of jam	=	$\frac{\text{mass of jam}}{\text{density of jam}} = \frac{\pi r^3 tdj}{b}$
Area of jam	=	area of circle of radius $r - \frac{1}{2} = \pi(r - \frac{1}{2})^2$
Thickness of jam	=	$\frac{\text{volume}}{\text{area}} = \frac{r^3 tdj}{b(r-\frac{1}{2})^2}$
Total thickness	=	thickness of half scone + thickness of jam + thickness of cream

Note

This work was not sponsored by a cream company.