On the perfect size for a pizza

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Abstract

We investigate the mathematical relationship between the size of a pizza and its ratio of topping to base in a median bite. We show that for a given recipe, it is not only the overall thickness of the pizza that is is affected by its size, but also this topping-to-base ratio.

1 Basic assumptions

Quantities of topping and base. We assume that the same pizza recipe is being used at all times, so that the total quantity of dough and total quantity of topping remain constant.

Shape. We assume the pizza is circular although, of course, no pizza is a perfect circle.

Edge. We assume that an edge is left around the pizza topping. Following some experiments, we assume that it is proportional to the thickness of the pizza—we prefer to leave a bigger edge when the base is thicker. Perhaps this is because the topping has further to fall off.

Median bite. We assume that more than half of the bites will come from the centre of the pizza, that is, a part not including the edge. This is a reasonable assumption for all but very small pizzas. Thus a median bite will be from a part of the pizza not including the edge.

2 The formula

Writing d for the constant volume of dough and t for the constant volume of topping, the ratio of topping to base in a median bite is:

$$\frac{t}{d}.\frac{r^6}{(r^3-15)^2}.$$

We can now compare the ratios in two different sizes of pizza, for example diameters of 14" and 11" (radius 7" and 5.5"). The formula tells us that a median bite from an 11" pizza has 10% more topping than a median bite from the 14" pizza.

Appendix: calculations

We use the following variables.

$$\begin{aligned} r &= \text{radius of pizza (half the diameter) in inches} \\ d &= \text{volume of dough (constant)} \\ t &= \text{volume of topping (constant)} \\ \alpha &= \text{scaling constant for the edge} \end{aligned}$$

We then have the following formulae.

$$\begin{aligned}
\text{Area of pizza base} &= \pi r^2 \\
\text{Thickness of pizza base} &= \frac{\text{volume of base}}{\text{area of base}} = \frac{d}{\pi r^2} \\
\text{Width of edge (to define the constant } \alpha) &= \text{constant} \times \text{thickness of base} = \frac{\alpha}{r^2} \\
\text{Area of topping} &= \pi\left(r - \frac{\alpha}{r^2}\right)^2 \\
\text{Thickness of topping} &= \frac{\text{volume of topping}}{\text{area of topping}} = \frac{t}{\pi\left(r - \frac{\alpha}{r^2}\right)^2} \\
\text{Ratio of topping to base in the middle} &= \frac{\text{thickness of topping}}{\text{thickness of base}} \\
&= \frac{t}{d}.\frac{r^2}{\left(r - \frac{\alpha}{r^2}\right)^2} \quad = \quad \frac{t}{d}.\frac{r^6}{(r^3-\alpha)^2}
\end{aligned}$$

Note on the constant α

We used the value $\alpha = 15$ in the main formula[1]. This was found experimentally. It provides the following sample edge widths[2]:

diameter of pizza (in inches)	width of edge (in mm)
10	15
11	13
12	11
13	9
14	8

[1] Technically this constant has absorbed an instance of d so should be measured in inches cubed.

[2] For some reason it seemed natural to measure diameters of pizzas in inches, but widths of edges in millimetres. We chose not to fight that urge. Note that as the pizza gets smaller, you have to pile your topping up higher and higher to keep the same amount of topping. It will be infinitely high when $r = \alpha^{\frac{1}{3}}$.

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