

## How I know my multiplication tables

In which I discover that the only multiplications I have memorised are  $6 \times 7$  and  $7 \times 8$ .

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Yes I am a mathematician:

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Many of my art students tell me that the thing that really put them off mathematics was having to memorise multiplication tables. I have great sympathy for this and personally never did memorise my multiplication tables. And yet I can multiply! I realised that I do it by working things out, with a small number of exceptions. I do it quickly enough that it is just as fast as having "memorised" it, with the advantage that I never had to sit down and memorise it. The only disadvantage I can see is that it's not innate enough for me to be able to multiply numbers bigger than 10 easily, and so I'll never be a human calculator: what a pity. Not.

Here's an honest analysis of how I do my times tables up to 9.

The first thing I've observed is that I can't multiply anything if I use the word "multiply" or "times". I have to turn "two times four" into "two fours" in my head first. For many situations I use a combination of visualisation, and oral/aural memory of how the words sound, just like when you remember the words of a song without having ever sat down and deliberately memorised them. I use this combination simultaneously, to cross check my answer.

I do all this in a split second, so if you watch me do it you won't notice it happening, unless you watch my eyes very carefully.

### General patterns

- I can double all the numbers up to 9, by a combination of practice of counting in 2's, visualising two rows of dots, and aural memory.
- I can square all the numbers up to 9, by a combination of visualising square grids (for the smaller ones) and familiarity
- I can multiply by 5 because it's the same as multiplying by 10 and dividing by 2 (or the other way round)
- I use a lot of commutativity ( $a \times b = b \times a$ )

$2 \times 2$  square

Higher multiples of 2: commutativity

$2 \times 3$  double

$3 \times 3$  square

Higher multiples of 3: commutativity

$2 \times 4$  double

$3 \times 4$  familiarity with a 3 by 4 grid

$4 \times 4$  square  
 $5 \times 4$  half of 40, and familiarity  
 $6 \times 4$  "six fours are twenty four" like a song, and  $20 + 4$ , and  $30 - 6$   
 $7 \times 4$  "seven fours are twenty eight" like a song, and double 14  
Higher multiples of 4: commutativity

$2 \times 5$  double  
 $3 \times 5$  counting in fives  
 $4 \times 5$  counting in fives  
 $5 \times 5$  square  
 $6 \times 5$  counting in fives  
 $7 \times 5$  half of 70, also halfway between 30 and 40  
Higher multiples of 5: commutativity

$2 \times 6$  double  
 $3 \times 6$  double 6 plus 6, and familiarity  
 $4 \times 6$  commutativity  
 $5 \times 6$  commutativity  
 $6 \times 6$  square  
Higher multiples of 6: commutativity

$2 \times 7$  double  
 $3 \times 7$  double 7 plus 7, and familiarity  
 $4 \times 7$  commutativity  
 $5 \times 7$  commutativity  
 $6 \times 7$  "six sevens are forty two" like a song  
 $7 \times 7$  square  
Higher multiples of 7: commutativity

$2 \times 8$  double  
 $3 \times 8$  double 8 plus 8, and familiarity  
 $4 \times 8$  double 16, also  $40 - 8$ , also familiarity  
 $5 \times 8$  half 8 times 10, also familiarity  
 $6 \times 8$  "six eights are forty eight" like a song, and  $40 + 8$   
 $7 \times 8$  "seven eights are fifty six" like a song  
 $8 \times 8$  square  
 $9 \times 8$  commutativity

$2 \times 9$  double  
 $3 \times 9$   $30 - 3$   
 $4 \times 9$   $40 - 4$   
 $5 \times 9$  half 9 times 10  
 $6 \times 9$  "six nines are fifty four" like a song, and  $60 - 6$   
 $7 \times 9$   $70 - 7$   
 $8 \times 9$   $80 - 8$   
 $9 \times 9$  square